Characteristics of Multiconductor, Asymmetric, Slow-Wave Microstrip Transmission Lines

TSUNG-CHENG MU, HIROYO OGAWA, MEMBER, IEEE, AND TATSUO ITOH, FELLOW, IEEE

***Abstract* —Spectral-domain technique has been applied to analyze multiconductor, asymmetric, slow-wave microstrip lines. It is observed that 1) the coupled slow-wave microstrip line on a two-layer substrate may have substantially different propagation constants for even and odd modes and 2) the slow-wave factor of an odd mode of coupled microstrip lines on a three-layer substrate may be equal to or larger than that of an even mode under appropriate conditions. This presents the flexibility to realize a large variety of passive components, such as directional couplers, phase shifters, and attenuators.**

I. Introduction

PLANAR SLOW-WAVE structures based on semiconductor substrates are currently of great technical interest. The first reason is that, with a growing interest in very high-speed digital integrated circuits, a thorough knowledge of the properties of various planar transmission lines on semiconductor substrates is essential in order to take full advantage of the inherent speed capability of the devices [1]. Second, the slow-wave phenomena have shown a potential to reduce the dimension of distributed components substantially so that the realization of novel integrated circuits for microwave frequency can be expected.

A number of analytical studies have been reported on several kinds of planar slow-wave structures [2]–[7]. Simplified parallel-plate structures were first examined [2], [3], and other studies based on a hybrid-mode approach have shown the applicability of several techniques to the analysis of MIS microstrip lines, MIS coplanar waveguides [5], [6]. However, no theoretical results based on a full-wave analysis have been reported on multiconductor, asymmetric, slow-wave transmission lines except the coupled microstrip lines on two-layer substrates [7]. These structures are expected to realize a wide variety of passive components such as directional couplers, phase shifters, and attenuators, and to have a large impact on monolithic microwave integrated circuits.

The purpose of this paper is to present a detailed analysis of multiconductor, asymmetric, slow-wave microstrip lines based on the spectral-domain technique [8]. Some interesting characteristics about two-layer and three-layer substrates have been discovered. The results are compared with the experimental and theoretical data [4] in the limiting case where the spacing between coupled microstrip lines becomes infinity.

Manuscript received March 17, 1986; revised May 16, 1986. This work was supported by U.S. Army Research Office Contract DAAG29-84-K-0076.

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IEEE Log Number 8610562.

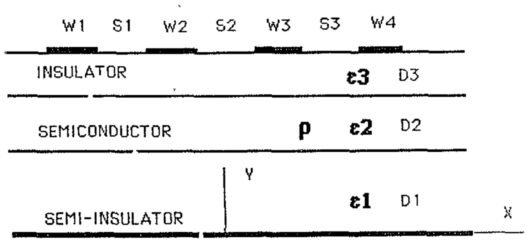

Fig. 1. Cross-sectional view of multiconductor asymmetric slow-wave microstrip lines.

II. Spectral-Domain Method

The structure in Fig. 1 can model the slow-wave structure of MIS configuration as well as of the Schottky contact type. In the present case, the doped region of a semiconductor substrate is treated as a dielectric layer with finite resistivity, which is included in the analysis by the complex permittivity for the layer. The reason why we can apply the present model to a Schottky structure is that most of the field lines are concentrated under and between strip conductors, and the field far away from the strip conductors has very little effect on propagation characteristics.

A simple method for formulating the dyadic Green's function in the spectral domain based on the transverse equivalent transmission line is adopted to two-layer and three-layer substrates. The following set of coupled equations are obtained with the same formulation process as in [8]:

$$\begin{aligned}\tilde{Z}_{zz}^{(i)}(\alpha,\beta)\tilde{J}_z(\alpha)+\tilde{Z}_{zx}^{(i)}(\alpha,\beta)\tilde{J}_x(\alpha)&=\tilde{E}_z(\alpha)\\ \tilde{Z}_{xz}^{(i)}(\alpha,\beta)\tilde{J}_z(\alpha)+\tilde{Z}_{xx}^{(i)}(\alpha,\beta)\tilde{J}_x(\alpha)&=\tilde{E}_x(\alpha)\end{aligned}\tag{1}$$

where $\tilde{Z}_{zz}^{(i)}$, $\tilde{Z}_{zx}^{(i)}$, $\tilde{Z}_{xz}^{(i)}$, and $\tilde{Z}_{xx}^{(i)}$ are the Green's impedance functions; i denotes the number of layers; and $\tilde{J}_z$, $\tilde{J}_x$, $\tilde{E}_z$, and $\tilde{E}_x$ are the Fourier transform of the z and x components of current densities and electric fields, respectively. Here, α is the Fourier transform variable and β is an unknown propagation constant. The closed-form Green's functions are derived in the Appendix for two ($i=2$) and three ($i=3$) layers.

The solution is then found by means of Galerkin's method in the spectral domain, where the current densities J_z and J_x are expanded in terms of complete sets of known basis functions [5]. After taking the Fourier transform of

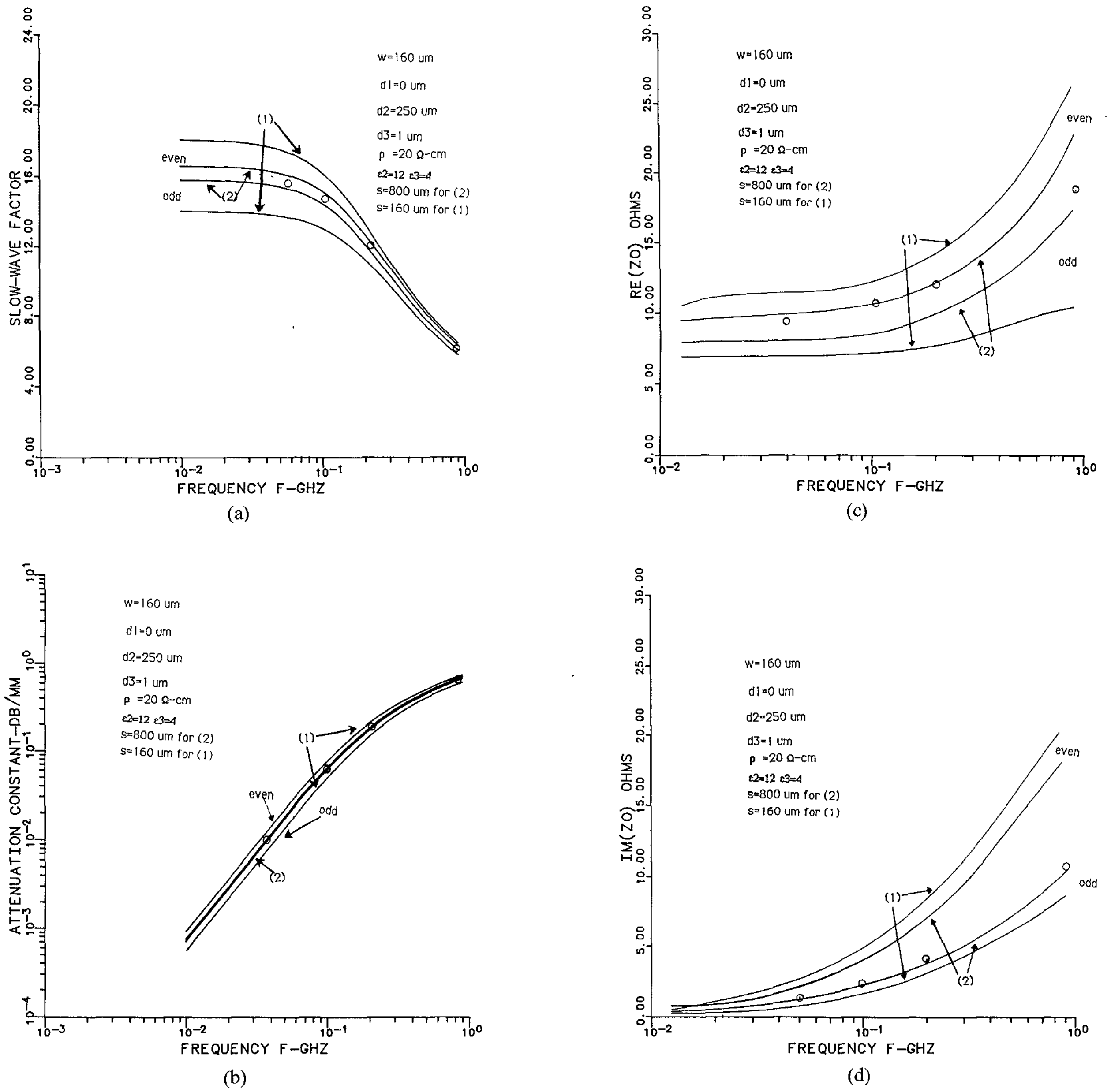

Fig. 2. (a) Slow-wave factor of coupled slow-wave microstrip lines (symmetric two-conductors) for two-layer substrates. (b) Attenuation constant of coupled slow-wave microstrip lines for two-layer substrates. (c) Real part of the characteristic impedance of coupled slow-wave microstrip lines for two-layer substrates. (d) Imaginary part of the characteristic impedance of coupled slow-wave microstrip lines for two-layer substrates.

these expressions, they are substituted into (1). Then the inner products of the resultant equations with each of the basis functions are formed. The homogeneous linear simultaneous equations are obtained and the right-hand side becomes identically zero by the inner product process. By equating the determinant of simultaneous equations to zero, the eigenvalue β is obtained.

The definition of characteristic impedance is based on the power transported along the planar lines and is written as follows [9]:

$$Z = \frac{\int\int E \times H^{*} \cdot z \, dx \, dy}{I_z \cdot I_z^{*}} \tag{2}$$

where I_z is the total current in the z direction and $\hat{z}$ is the z-directed unit vector. This definition can apply even if more than two strips or asymmetric cases are presented in the problem.

III. Numerical Results and Discussion

A. Two-Layer Structure

The characteristics of the symmetric two-strip configuration are first calculated with the analytical procedures described above. Fig. 2 shows the frequency dependence of the slow-wave factor, attenuation constants, and characteristic impedances for two different values of strip spacing in two-layer structures. The experimental results by Hasegawa *et al.* [4] are included for comparison. When the spacing between the coupled microstrip lines becomes infinity,

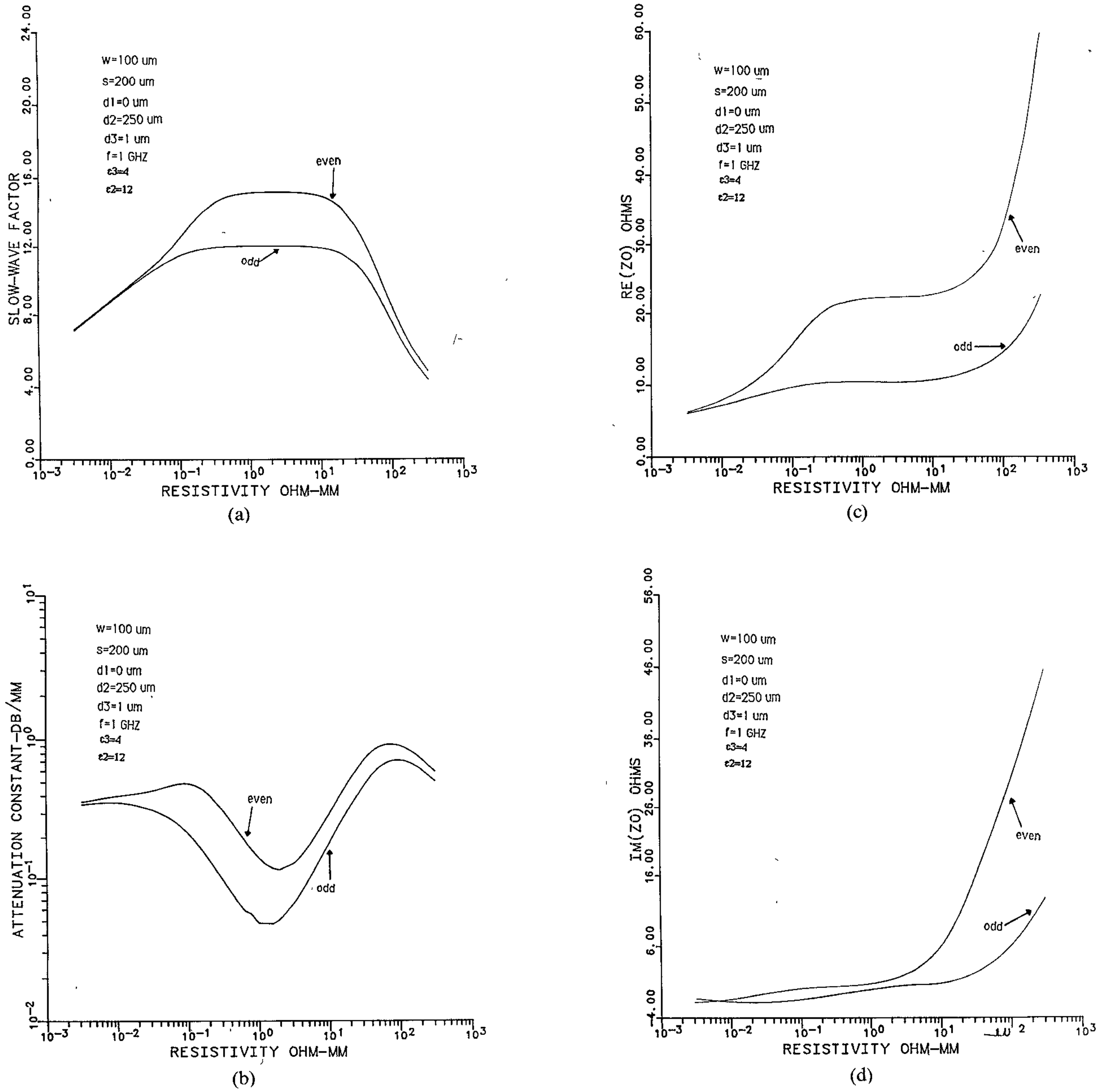

Fig. 3. (a) Slow-wave factor of coupled microstrip lines for two-layer substrates versus resistivity. (b) Attenuation constant of coupled microstrip lines for two-layer substrates versus resistivity. (c) Real part of characteristic impedance of coupled microstrip lines for two-layer substrates versus resistivity. (d) Imaginary part of characteristic impedance of coupled microstrip lines for two-layer substrates versus resistivity.

both even- and odd-mode propagation constants approach the value of the single microstrip line.

From Fig. 2, it is noticed that the slow-wave factors, attenuation constants, and characteristic impedances are substantially different from those of the uncoupled line. In the even mode, more field lines pass through the conductive layer than in the single microstrip line. In the odd mode, a much smaller part of the field passes through this layer because a substantial portion of the field lines connects the two strips. Therefore, the slow-wave factor for the even mode is very different from that for the odd mode.

Fig. 3 shows the propagation characteristics as a function of the substrate resistivity of the doped region, which are similar to those predicted for MIS microstrip line [10]. There are basically three operating regions: the slow-wave region, the skin-effect region, and the lossy-dielectric region. Both the slow-wave factor and the characteristic impedance in the slow-wave region are almost constant and optimum resistivity exists for the slow-wave propagation.

The present program can also handle asymmetric situations. Fig. 4 shows the propagation characteristics of two-strip, asymmetric, slow-wave microstrip lines. Although the propagation constant changes by only 10 percent, the real part of the characteristic impedance changes by more than 50 percent. This characteristic can be applied to adjust the impedance level of the circuit.

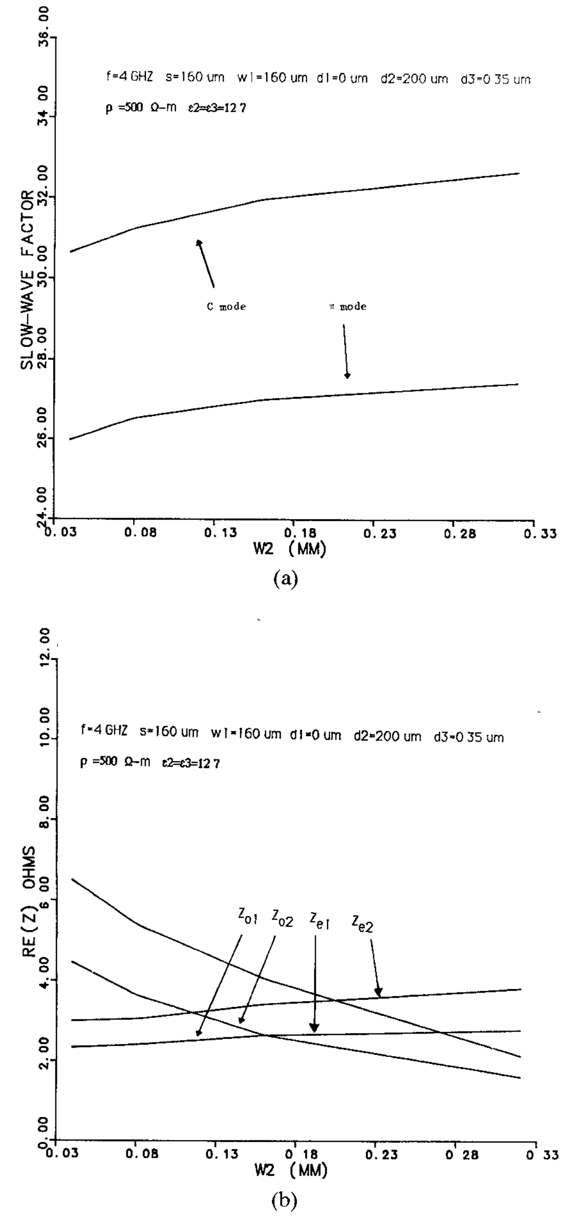

Fig. 4. (a) Slow-wave factor of coupled asymmetric slow-wave microstrip lines (two conductors) for two-layer substrates. (b) Real part of the characteristic impedance of coupled asymmetric slow-wave microstrip lines for two-layer substrates. Z_{e1} and Z_{e2} are the characteristic impedances of C mode and Z_{o1} and Z_{o2} that of π mode.

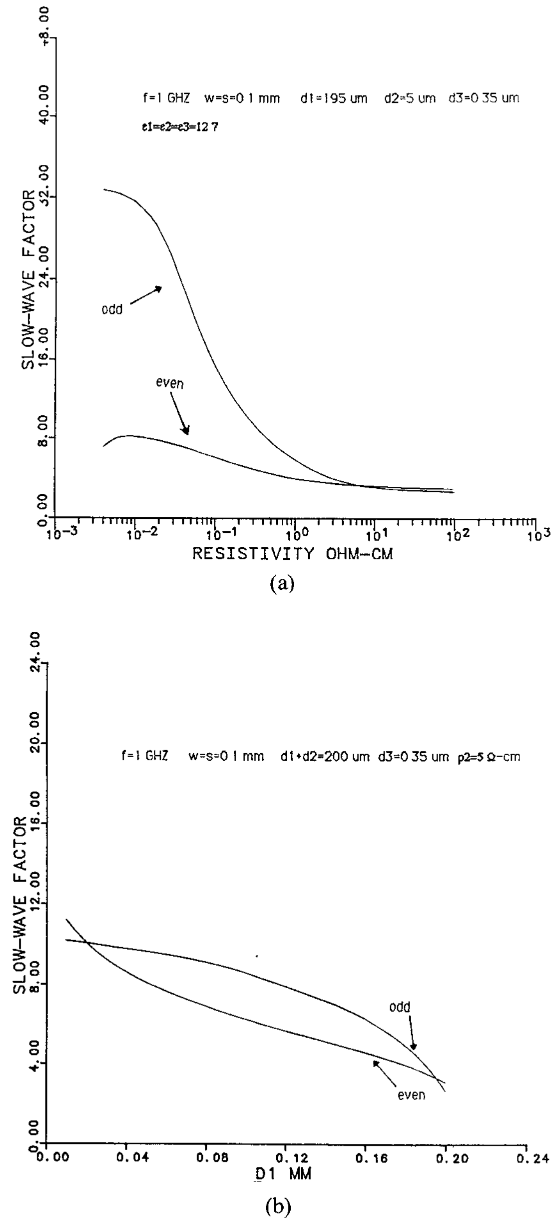

Fig. 5. (a) Slow-wave factor of coupled slow-wave microstrip lines (symmetric two conductors) for three-layer substrates versus resistivity. (b) Slow-wave factor of coupled slow-wave microstrip lines for three-layer substrates versus $d1$.

B. Three-Layer Structure

Since the three-layer substrate is similar to the cross section of a GaAs MESFET, it is more practical and important to study the characteristics of such a structure. A very interesting phenomenon about the slow-wave factor has been found in this structure. Under some conditions, the slow-wave factor of the odd mode is larger than that of the even mode. In the conventional coupled microstrip lines, as well as in the coupled slow-wave microstrip lines on two-layer substrates [7], the propagation constant of the even mode is always larger than that of the odd mode because of the field distribution [11].

This new phenomenon can be explained in Fig. 5. Fig. 5(a) shows the slow-wave factor versus resistivity. Fig. 5(b) shows the slow-wave factor for different values of thickness of semi-insulating layers. When the resistivity is high, corresponding to lossless coupled microstrip lines, the propagation constant of the even mode is larger than that of the odd mode. However, when the resistivity becomes smaller, the slow-wave factor of the odd mode increases more than that of the even mode in the three-layer structure. The origin of this particular characteristic comes from the energy transfer across the interface between the lossy layer and insulating layer. In the even mode, the electric field tends to penetrate into the whole substrate. However, the lossy layer in the three-layer structure is usually very thin. The major portion of electric energy will be stored in both thin insulating and thick semi-insulating layers. In contrast to this, most of the electric energy is

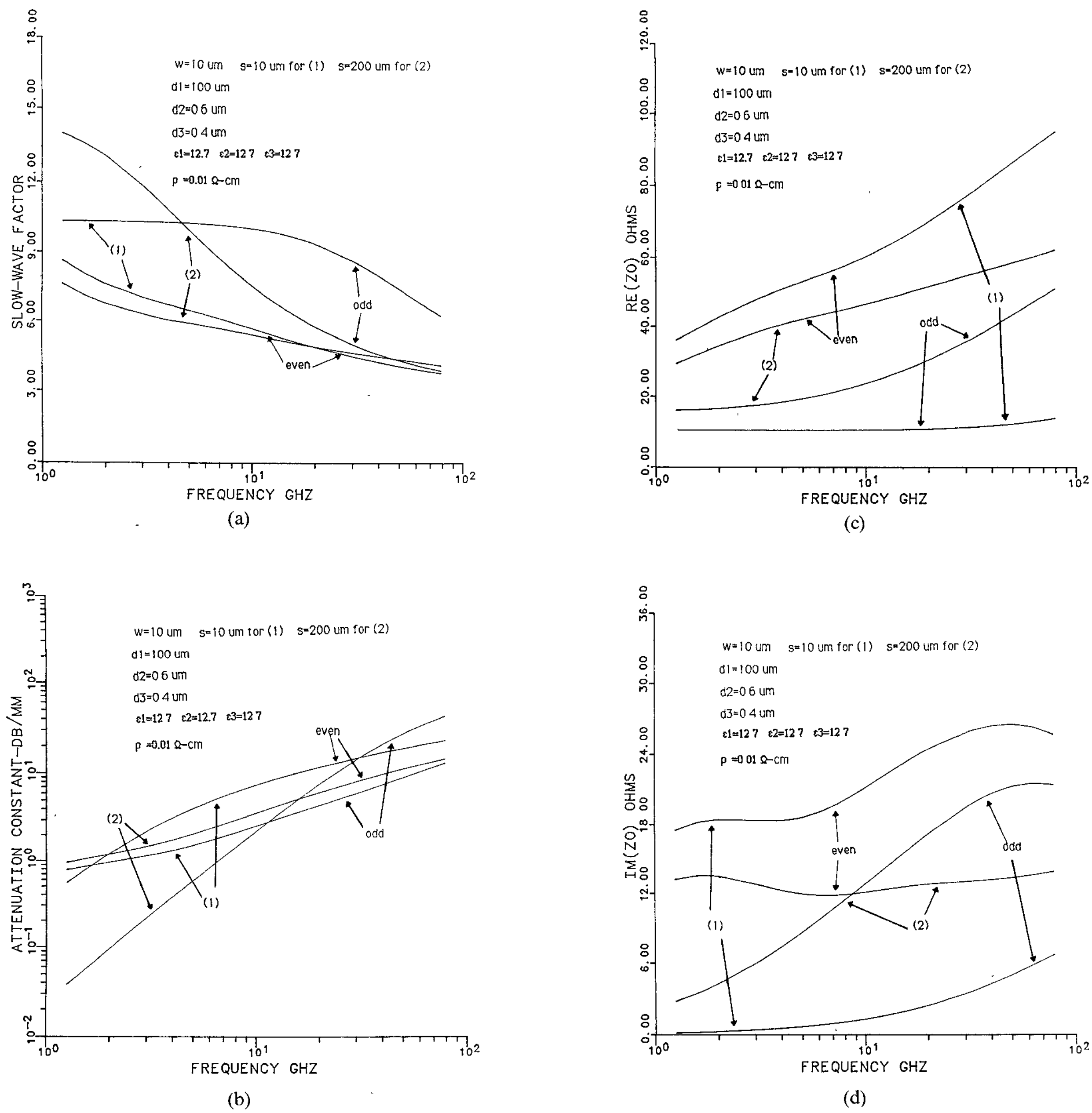

Fig. 6. (a) Slow-wave factor of coupled slow-wave microstrip lines for three-layer substrates. (b) Attenuation constant of coupled slow-wave microstrip lines for three-layer substrates. (c) Real part of the characteristic impedance of coupled slow-wave microstrip lines for three-layer substrates. (d) Imaginary part of characteristic impedance of coupled slow-wave microstrip lines for three-layer substrates.

stored in the thin insulating layer in the two-layer structure. In the odd mode, the electric field is concentrated between two strips. Therefore, the major part of the electric energy will be stored in the thin insulating layer and the slow-wave factor can be increased by the energy transfer between the thin insulating layer and the lossy layer. The slow-wave factor of the odd mode becomes greater than that of the even mode.

Two intersection points can be seen in Fig. 5(b). When the semi-insulating layer become very thin, corresponding to the two-layer structure, the slow-wave factor of the even mode becomes larger than that of the odd mode. On the other hand, when the semi-insulating layer becomes thick enough, the propagation constant of the odd mode will be smaller than that of the even mode, as in conventional coupled microstrip lines. Fig. 5 shows a useful feature of coupled slow-wave microstrip lines. The directional coupler with the same propagation constant of even and odd modes can be realized with parameters corresponding to these intersection points.

At the fixed resistivity of 0.01 Ω·cm, the three-layer structure is studied by varying the frequency. Fig. 6 shows the numerical results of slow-wave factors, attenuation constants, and characteristic impedances for two different values of spacing versus frequency on three-layer substrates. It is observed that the slow-wave factor of the odd mode is larger than that of the even mode for most of the frequency range.

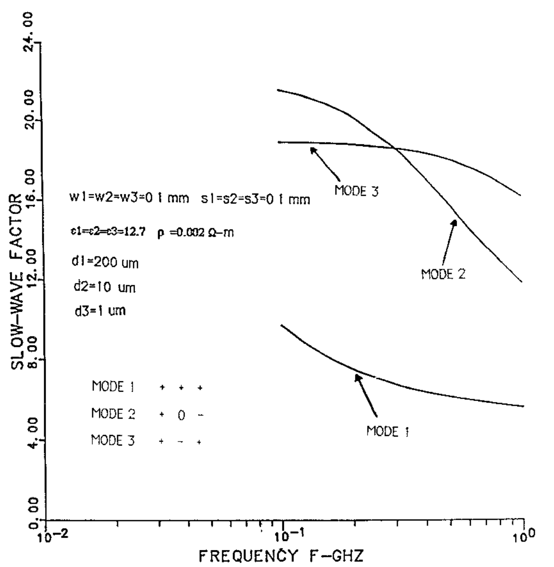

Fig. 7. Slow-wave factor of three-conductor symmetric slow-wave microstrip lines for three-layer substrates.

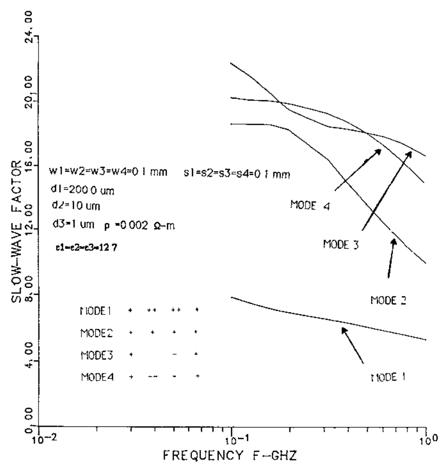

Fig. 8 Slow-wave factor of four-conductor symmetric slow-wave microstrip lines for three-layer substrates

Finally, the behavior of the propagation characteristics for a three-conductor and a four-conductor on three-layer substrates is also investigated. The calculated results for symmetric three-conductor and four-conductor configurations are shown in Figs. 7 and 8, respectively. The symbols $+$ and $-$ are used to indicate the direction of J_z. The meaning of $++$ is that the amplitude of the conductor is substantially larger than that of $+$. The relationship between $--$ and $-$ is the same. As in the coupled slow-wave microstrip-line structures, the all-positive potential situation has the lowest propagation constant among all the fundamental modes.

IV. Conclusions

The characteristics of multiconductor, asymmetric, slow-wave microstrips have been investigated thoroughly with the spectral-domain technique. Characteristics of these structures have been studied for different values of structural parameters such as thickness and resistivity of the doped semiconductor layer. The results obtained are as follows.

1) The coupled slow-wave microstrip line (symmetric two conductors) on two-layer substrate has a substantially different slow-wave factor for even and odd modes.

2) In a two-strip, asymmetric, slow-wave microstrip line on two-layer substrate, the characteristic impedance can be adjusted significantly with only a small variation of the slow-wave factor.

3) The coupled slow-wave microstrip line (symmetric two conductors) on a three-layer substrate has the unique characteristic that the slow-wave factor of the odd mode can be equal to or larger than that of the even mode.

4) In the symmetric three- or four-conductor slow-wave microstrip line, the all-positive potential mode has the lowest propagation constant among all the fundamental modes.

5) Since the slow-wave microstrip lines described here have structure similar to that of GaAs MESFET, they may be advantageously used in realizing physically small passive components such as directional couplers and phase shifters for MMIC.

Appendix

According to the transverse resonance method, the following Green's impedance functions are obtained:

$$\tilde{Z}_{zz}^{(j)}(\alpha,\beta)=\frac{\beta^2}{\alpha^2+\beta^2}\tilde{Z}^{ej}+\frac{\alpha^2}{\alpha^2+\beta^2}\tilde{Z}^{hj} \tag{A1}$$

$$\tilde{Z}_{zx}^{(j)}(\alpha,\beta)=\tilde{Z}_{xz}^{(j)}(\alpha,\beta)=\frac{\alpha\beta}{\alpha^2+\beta^2}\left(\tilde{Z}^{ej}-\tilde{Z}^{hj}\right) \tag{A2}$$

$$\tilde{Z}_{xx}^{(j)}(\alpha,\beta)=\frac{\alpha^2}{\alpha^2+\beta^2}\tilde{Z}^{ej}+\frac{\beta^2}{\alpha^2+\beta^2}\tilde{Z}^{hj}. \tag{A3}$$

A. Two-Layer Structure ($j=2$)

$$(\tilde{Z}^{e2})^{-1}=Y_{\mathrm{TM4}}+Y_{\mathrm{TM3}}\frac{Y_{\mathrm{TM2}}\coth\gamma_2 d_2+Y_{\mathrm{TM3}}\tanh\gamma_3 d_3}{Y_{\mathrm{TM3}}+Y_{\mathrm{TM2}}\coth\gamma_2 d_2\tanh\gamma_3 d_3} \tag{A4}$$

$$(\tilde{Z}^{h2})^{-1}=Y_{\mathrm{TE4}}+Y_{\mathrm{TE3}}\frac{Y_{\mathrm{TE2}}\coth\gamma_2 d_2+Y_{\mathrm{TE3}}\tanh\gamma_3 d_3}{Y_{\mathrm{TE3}}+Y_{\mathrm{TE2}}\coth\gamma_2 d_2\tanh\gamma_3 d_3} \tag{A5}$$

$Y_{\mathrm{TM}i}=\dfrac{j\omega\epsilon_0\epsilon_i}{\gamma_i}$, characteristic admittance for TM mode in ith region,

$Y_{\mathrm{TE}i}=\dfrac{\gamma_i}{j\omega\mu_0}$, characteristic admittance for TE mode in ith region,

$\gamma_i^2=\alpha^2+\beta^2-\epsilon_i k_0^2$, propagation constant in the y direction in the ith region.

B. Three-Layer Structure $(j=3)$

$$(\tilde{Z}^{e3})^{-1}=Y_{TM4}+Y_{TM3}\frac{Y_{TM2}(Y_{TM1}\coth\gamma_1 d_1+Y_{TM2}\tanh\gamma_2 d_2)+Y_{TM3}\tanh\gamma_3 d_3(Y_{TM2}+Y_{TM1}\coth\gamma_1 d_1\tanh\gamma_2 d_2)}{Y_{TM3}(Y_{TM2}+Y_{TM1}\coth\gamma_1 d_1\tanh\gamma_2 d_2)+Y_{TM2}\tanh\gamma_3 d_3(Y_{TM1}\coth\gamma_1 d_1+Y_{TM2}\tan\gamma_2 d_2)} \quad (A6)$$

$$(\tilde{Z}^{h3})^{-1}=Y_{TE4}+Y_{TE3}\frac{Y_{TE2}(Y_{TE1}\coth\gamma_1 d_1+Y_{TE2}\tanh\gamma_2 d_2)+Y_{TE3}\tanh\gamma_3 d_3(Y_{TE2}+Y_{TE1}\coth\gamma_1 d_1\tanh\gamma_2 d_2)}{Y_{TE3}(Y_{TE2}+Y_{TE1}\coth\gamma_1 d_1\tanh\gamma_2 d_2)+Y_{TE2}\tanh\gamma_3 d_3(Y_{TE1}\coth\gamma_1 d_1+Y_{TE2}\tanh\gamma_2 d_2)}. \quad (A7)$$

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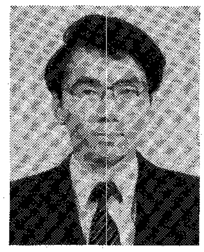

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